Yang-Mills origin of gravitational symmetries

Phys. Rev. Lett. 113, 231606

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EMFCSC, 2015

Outline

Introduce Gravity as the square of Yang-Mills
- String Theory origin

Towards a correspondance
- Multiplets, Global symmetries, Amplitudes and Solutions

Local symmetries
- Squaring
- Linear Supergravity and Linear Yang-Mills
- The dictionary

Comments and further work

$D = 10$ Type II from SYM

The massless open-string states consist of a vector and a spinor: (A_μ, χ). Using the $SO(8)$ tensor products:

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\begin{aligned}
\mathbf{8_v} \otimes \mathbf{8_v} &= \mathbf{35_v} \oplus \mathbf{28} \oplus \mathbf{1} \quad , \quad & \mathbf{8_v} \otimes \mathbf{8_s} &= \mathbf{56_c} \oplus \mathbf{8_c} \\
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(A_\mu, \chi^+) \otimes (A_\nu, \chi^-) = \; & (g_{\mu\nu} \oplus B_{\mu\nu} \oplus \phi) + (\psi^-_\mu \oplus \chi^+ \oplus \psi^+_\mu \oplus \chi^-) \\
& + (C_{\mu\nu\rho} \oplus A_\mu)
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- **Global symmetries**: Construct the U-duality group of the supergravity theory from the corresponding SYM parts. (1502.05359)
- **Amplitudes**: Calculating gravity scattering amplitudes using the double-copy formalism. (Bern, Carrasco, Johansson 0805.3993)
- **Solutions**: Schwarzschild is the double copy of the point Coulomb solution. (Monteiro, O'Connell, White 1410.0239)

$(\mathcal{N}_L = 1) \otimes (\mathcal{N}_R = 0)$ gives new-minimal $\mathcal{N} = 1$

Off-shell multiplets

$$V^i = (A^i_\mu, \chi^i, D^i)$$
$$4 + 4$$

↓ $A^{i\prime}_\nu$ $3 + 0$

$$\varphi_\nu = (Z_{\mu\nu}, \psi_\nu, V_\nu)$$
$$12 + 12$$

Gravity transformations

By writing the metric as:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

the diffeos:

$$x'^{\mu} = x^{\mu} + \kappa\xi^{\mu}$$

give the "gauge" transformation:

$$\delta g_{\mu\nu} = \delta h_{\mu\nu} = \kappa\nabla_{\mu}\xi_{\nu} + \kappa\nabla_{\nu}\xi_{\mu} = \kappa(\partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu}) + \mathcal{O}(\kappa^2)$$

and thus to linear order:

$$\delta h_{\mu\nu} = \partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu}$$

Yang-Mills transformations

By writing the gauge parameter as:

$$\vartheta^i(x) = \theta^i + g\sigma^i(x)$$

the Yang-Mills transformation becomes:

$$\delta A^i_\mu = \partial_\mu \vartheta^i - g f^i{}_{jk} \vartheta^j A^k_\mu = g(\partial_\mu \sigma^i - f^i{}_{jk}\theta^j A^k_\mu) + \mathcal{O}(g^2)$$

and thus to linear order:

$$\delta A^i_\mu = \partial_\mu \sigma^i - f^i{}_{jk}\theta^j A^k_\mu$$

The goal

The goal is to reproduce the gravity gauge transformations:

$$\begin{aligned}
\delta Z_{\mu\nu} &= \partial_\mu \alpha_\nu + \partial_\nu \beta_\nu \ \text{ where } Z_{\mu\nu} \equiv h_{\mu\nu} + B_{\mu\nu} \\
\delta \psi_\nu &= \partial_\nu \eta \\
\delta V_\nu &= \partial_\nu \Lambda
\end{aligned} \tag{1}$$

from the Yang-Mills gauge transformations:

$$\begin{aligned}
\delta A^i_\mu &= \partial_\mu \sigma^i - f^i{}_{jk} \theta^j A^k_\mu \\
\delta \chi^i &= -f^i{}_{jk} \theta^j \chi^k \\
\delta D^i &= -f^i{}_{jk} \theta^j D^k \\
\delta A^{i'}_\nu &= \partial_\nu \sigma^{i'} - f^{i'}{}_{j'k'} \theta^{j'} A^{k'}_\nu
\end{aligned} \tag{2}$$

The field dictionary

The field dictionary is:

$$\begin{aligned} Z_{\mu\nu} &\equiv A^{i}_{\mu} \star \Phi_{ii'} \star A^{i'}_{\nu} \\ \psi_{\nu} &\equiv \chi^{i} \star \Phi_{ii'} \star A^{i'}_{\nu} \\ V_{\nu} &\equiv D^{i} \star \Phi_{ii'} \star A^{i'}_{\nu} \end{aligned} \quad (3)$$

where the spectator field transforms in the bi-adoint.

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where the spectator field transforms in the bi-adoint. **The convolution is crucial in the derivation as it does not obey the Leibniz rule:**

$$(a \star b)(x) = \int a(y) b(x-y) d^4 y \quad (4)$$

$$\partial_\mu (a \star b) = \partial_\mu a \star b = a \star \partial_\mu b$$

The transformations

By varying the Yang-Mills parts, the gravity transformations become:

$$\begin{aligned} \delta Z_{\mu\nu} &= \partial_\mu(\sigma^i \star \Phi_{ii'} \star A_\nu^{i'}) + \partial_\nu(A_\mu^i \star \Phi_{ii'} \star \sigma^{i'}) \\ \delta \psi_\nu &= \partial_\nu(\chi^i \star \Phi_{ii'} \star \sigma^{i'}) \\ \delta V_\nu &= \partial_\nu(D^i \star \Phi_{ii'} \star \sigma^{i'}) \end{aligned} \tag{5}$$

The parameter dictionary

Thus the parameter dictionary is:

$$\begin{aligned}
\alpha_\nu &\equiv \sigma^i \star \Phi_{ii'} \star A_\nu^{i'} \\
\beta_\mu &\equiv A_\mu^i \star \Phi_{ii'} \star \sigma^{i'} \\
\eta &\equiv \chi^i \star \Phi_{ii'} \star \sigma^{i'} \\
\Lambda &\equiv D^i \star \Phi_{ii'} \star \sigma^{i'}
\end{aligned} \tag{6}$$

Comments

What else works:

- Poincare transformations work trivially.
- Supersymmetry transformations follow in a nice way.

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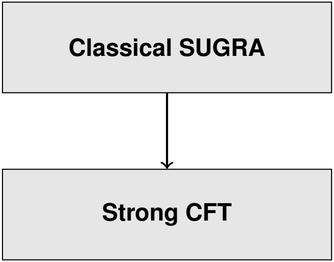
- Poincare transformations work trivially.
- Supersymmetry transformations follow in a nice way.

Two major improvements:

- Extend to $\mathcal{N}_R = 1$
- Extend to higher orders were non-linear effects start to appear.

Future work



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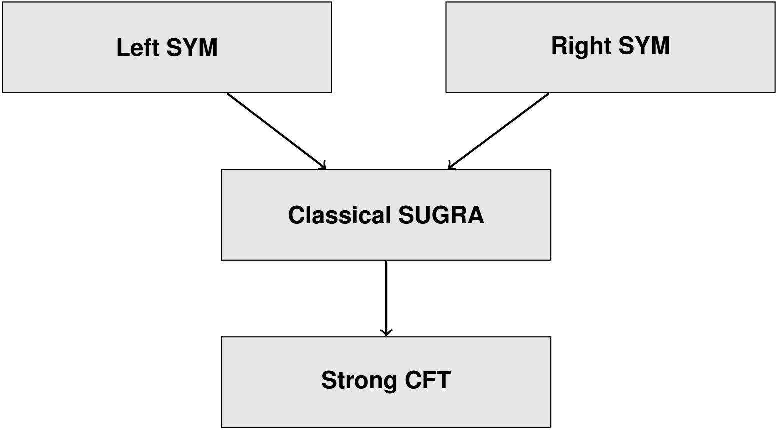

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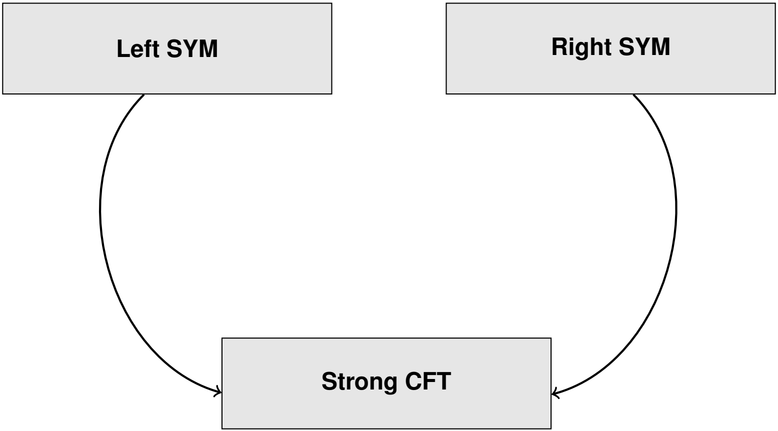

THANK YOU